Math 1261 Calculus I -- Test 1 Review Sheet
Preview, Chapter 1, but primarily Sections 2.1-2.7
Topics: Functions, Limits, Continuity

• Be able to state
  o definition of function
  o definition for operations (sum, difference, product, quotient) on functions
  o definition for composition of functions
  o the \( \varepsilon-\delta \) definition of the two-sided limit
  o precise definitions for one-sided limits, infinite limits, and limits at infinity
  o the definition of "A function \( f \) is continuous at a real number \( a \)"
  o the definition of "A function \( f \) is continuous on an interval \( I \)"
  o definition of horizontal asymptote
  o definition of vertical asymptote
  o definition of oblique asymptote
  o The Squeeze Theorem (also known as The Sandwich Theorem)
  o The Intermediate Value Theorem (IVT)

• Functions
  o Define in words
  o Basic Notation: \( f(x) \) means the partner the function \( f \) pairs with \( x \) when \( x \) is in the domain of \( f \)
  o Operations: sum, difference, product, quotient, composition
  o Basic Terminology: domain, codomain, range, value, etc.

• Limits
  o intuitive approach (from a graph, from a table)
  o stating \( \varepsilon-\delta \) definition
  o using \( \varepsilon-\delta \) definition
    • "graphs" to visualize \( \varepsilon-\delta \) pairs
    • "proofs" for linear functions
  o one-sided limits
  o infinite limits and vertical asymptotes
  o limits at infinity and horizontal asymptotes
  o using the Squeeze (Sandwich) Theorem
  o algebraic rewrite ("cancelling away problems")
    • factoring, conjugates, other algebraic techniques
  o using limit laws (one at a time) to evaluate a limit
  o providing sample graphs (when possible) to illustrate various conditions

• Continuity
  o definitions for continuous at a point and continuous on an interval
  o one-sided continuity
  o continuity for combinations (sum, product, difference, quotient, composition) of functions
  o discontinuous (not continuous)
  o determine points of discontinuity of a function (graphs or analytically)
  o determine intervals of continuity for a function (graphs or analytically)
  o state and apply the Intermediate Value Theorem
  o providing sample graphs (when possible) to illustrate various conditions

• Operations on Functions
  o Sum: Given \( f \) and \( g \) are functions, the sum of \( f \) and \( g \), denoted \( f+g \), is the function with formula description 
    \( (f+g)(x) = f(x) + g(x) \). [Note that \( \text{dom}(f+g) = \text{dom}(f) \cap \text{dom}(g) \)]
  o Difference: Given \( f \) and \( g \) are functions, the difference of \( f \) and \( g \), denoted \( f-g \), is the function with formula description 
    \( (f-g)(x) = f(x) - g(x) \). [Note that \( \text{dom}(f-g) = \text{dom}(f) \cap \text{dom}(g) \)]
  o Product: Given \( f \) and \( g \) are functions, the product of \( f \) and \( g \), denoted \( fg \), is the function with formula description 
    \( (fg)(x) = f(x)g(x) \). [Note that \( \text{dom}(fg) = \text{dom}(f) \cap \text{dom}(g) \)]
  o Quotient: Given \( f \) and \( g \) are functions, the quotient of \( f \) and \( g \), denoted \( f/g \), is the function with formula description 
    \( (f/g)(x) = f(x)/g(x) \). [Note that \( \text{dom}(f/g) = \text{dom}(f) \cap \text{dom}(g) \setminus \{x \mid g(x) = 0\} \)]
  o Composition: If \( f \) and \( g \) are functions, the composition of \( f \) with \( g \), denoted \( f \circ g \), is the function with formula description 
    \( (f \circ g)(x) = f(g(x)) \). [Note that \( \text{dom}(f \circ g) = \{x \mid x \text{ is in } \text{dom}(g) \text{ and } g(x) \text{ is in } \text{dom}(f)\} \)]
  o Note: Throughout – when \( h \) is a function, the notation \( \text{dom}(h) \) means “the domain of function \( h \)” and for sets \( A \) and \( B \),
    the notation \( A \cap B \) is the intersection of sets \( A \) and \( B \) meaning the elements in both sets, \( A \cap B = \{x \mid x \text{ is in } A \text{ and } x \text{ is in } B\} \), and \( A \setminus B \) is the set difference, that is the set of objects that are in \( A \) and not in \( B \) so that \( A \setminus B = \{x \mid x \text{ is in } A \text{ and } x \text{ is not in } B\} \).
Other Definitions and Theorems

Function: See class handout

\(\varepsilon-\delta\) definition of the two-sided limit: Let \(f\) be a real-valued function defined on an open interval containing the real number \(a\) except possibly at the real number \(a\) itself. Then \(\lim_{x \to a} f(x) = L\), where \(L\) is a real number, means that for every real number \(\varepsilon\) with \(\varepsilon > 0\), there is a real number \(\delta > 0\) such that for all real numbers \(x\) in the domain of \(f\), if \(0 < |x - a| < \delta\) then \(|f(x) - L| < \varepsilon\).

Mathematical definition of infinite limit (for \(L = +\infty\)): Let \(f\) be a real-valued function defined on an open interval containing the real number \(a\) except possibly at the real number \(a\) itself. Then \(\lim_{x \to a} f(x) = +\infty\) means that for every real number \(M\) with \(M > 0\), there is a real number \(\delta > 0\) such that for all real numbers \(x\) in the domain of \(f\), if \(0 < |x - a| < \delta\) then \(f(x) > M\).

Mathematical definition of infinite limit (for \(L = -\infty\)): Let \(f\) be a real-valued function defined on an open interval containing the real number \(a\) except possibly at the real number \(a\) itself. Then \(\lim_{x \to a} f(x) = -\infty\) means that for every real number \(M\) with \(M < 0\), there is a real number \(\delta > 0\) such that for all real numbers \(x\) in the domain of \(f\), if \(0 < |x - a| < \delta\) then \(f(x) < M\).

Mathematical definition of limit at infinity (for \(a = +\infty\)): Let \(f\) be a real-valued function defined on an open interval of the form \((a, \infty)\) for some real number \(a\). Then \(\lim_{x \to +\infty} f(x) = L\), where \(L\) is a real number, means that for every real number \(\varepsilon\) with \(\varepsilon > 0\), there is a real number \(M\) with \(M > 0\) such that for all real numbers \(x\) in the domain of \(f\), if \(x > M\) then \(|f(x) - L| < \varepsilon\).

Mathematical definition of limit at infinity (for \(a = -\infty\)): Let \(f\) be a real-valued function defined on an open interval of the form \((-\infty, a)\) for some real number \(a\). Then \(\lim_{x \to -\infty} f(x) = L\), where \(L\) is a real number, means that for every real number \(\varepsilon\) with \(\varepsilon > 0\), there is a real number \(M\) with \(M < 0\) such that for all real numbers \(x\) in the domain of \(f\), if \(x < M\) then \(|f(x) - L| < \varepsilon\).

Continuity of a function \(f\) at a real number \(a\): Let \(f\) be a real-valued function defined on an open interval containing the real number \(a\). Then \(f\) is continuous at the real number \(a\) if and only if (1) \(f(a)\) is defined, (2) \(\lim_{x \to a} f(x)\) exists, and (3) \(\lim_{x \to a} f(x) = f(a)\).

\(\varepsilon-\delta\) definition of continuity of a function \(f\) at the real number \(a\): Let \(f\) be a real-valued function defined on an open interval containing the real number \(a\). Then \(f\) is continuous at \(a\) means that for every real number \(\varepsilon\) with \(\varepsilon > 0\), there is a real number \(\delta > 0\) such that for all real numbers \(x\) in the domain of \(f\), if \(0 < |x - a| < \delta\) then \(|f(x) - f(a)| < \varepsilon\).

Horizontal Asymptote: A horizontal line \(y = L\), where \(L\) is a real number, is a horizontal asymptote (HA) of a real-valued function \(f\) if and only if at least one of the following holds (1) \(\lim_{x \to +\infty} f(x) = L\) (2) \(\lim_{x \to -\infty} f(x) = L\).

Oblique Asymptote: A line \(y = mx + b\), where \(m\) and \(b\) are real numbers and \(m \neq 0\), is an oblique asymptote (OA) of a real-valued function \(f\) if and only if at least one of the following holds (1) \(\lim_{x \to +\infty} [f(x) - (mx + b)] = 0\) (2) \(\lim_{x \to -\infty} [f(x) - (mx + b)] = 0\).

Vertical Asymptote: A vertical line \(x = a\), where \(a\) is a real number, is a vertical asymptote (VA) of a real-valued function \(f\) if and only if at least one of the following holds (1) \(\lim_{x \to a^+} f(x) = +\infty\) (2) \(\lim_{x \to a^-} f(x) = -\infty\) (3) \(\lim_{x \to a^+} f(x) = +\infty\) (4) \(\lim_{x \to a^-} f(x) = -\infty\).

The Squeeze Theorem: If \(f\), \(g\), and \(h\) are real-valued functions for which \(f(x) \leq g(x) \leq h(x)\) for all \(x\) in an open interval containing a real number \(a\) except possibly at \(x = a\) and there is a real number \(L\) for which \(\lim_{x \to a} f(x) = L\) and \(\lim_{x \to a} h(x) = L\), then \(\lim_{x \to a} g(x) = L\).

The Intermediate Value Theorem: If \(a\) and \(b\) are real numbers with \(a < b\) for which \(f\) is a continuous real-valued function on the closed interval \([a,b]\) and \(f(a) \neq f(b)\), then for any real number \(N\) between \(f(a)\) and \(f(b)\) there is at least one real number \(c\), where \(c\) is contained in the open interval \((a,b)\), for which \(f(c) = N\).